ABSTRACT

Great difficulty can be met in justifying of many high and steep slopes in residual soils, even under heavy rainfall, and yet some slopes fail during heavy rainfall and some after remaining stable for many years. It is now clear that unsaturated zone plays a crucial role in the slope stability as it is the link between surface and groundwater. The objective of this paper is to develop a solution where the unsaturated flow equation and the formulation of appropriate effective stress equation and parameters for such soils can be solved simultaneously.

INTRODUCTION

The analysis of water movement in unsaturated zone can be performed by a the quasi-analytical approach or a computer based numerical approach. In this paper the numerical approach will be adopted. In general, the differential equation representing flow in one or more directions are solved by finite difference techniques using high digital computer. The numerical approach has contributed in the understanding and assessment of the unsaturated flow behaviour.

The use of saturated shear strength parameters in slope stability analysis has recently come under very close scrutiny by researchers. Slope failures in location where groundwater table is deep or where failure surface is shallow need a different treatment to that on normally used methods.
Bishop (1960) and Fredlund (1987) have established relationships between factor of safety with respect to the sliding of slopes. In this paper the discussion is limited to a single aspect: that the water table is initially at a fixed level and infiltration through the soil surface is then permitted until the profile is saturated.

COMPUTER BASED NUMERICAL SOLUTIONS

By establishing an appropriate finite difference solution of the flow equation, it is possible to solve a wide range of unsaturated flow systems in one or more dimensions. Lack of homogeneity in the profile and variations in initial water content can now be readily handled. In addition, the hysteresis complications introduced by intermittency in the surface flow pattern present no basic problems other than those of more exacting programming and possible computer storage insufficiency. In this paper it has been assumed that the movement of air in the porous medium is of insignificance and can be neglected.

The particular numerical approach summarized in this paper is that used by Awadalla (1988). The pressure head form of the flow equation has been used in the analysis since this has decided advantages where saturated conditions exist in some part of the system and where layering occurs in the profile (the pressure head must be continuous across an interface). For vertical isothermal flow in a homogeneous porous medium,

\[ C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left( K(h) \frac{\partial h}{\partial z} \right) + \frac{\partial K(h)}{\partial z} \quad (1) \]

where
- \( c(h) \) is the specific water capacity (cm)
- \( h \) pressure head (cm)
- \( k \) hydraulic conductivity (cm/min)
- \( z \) vertical ordinate measured positively upwards (cm)

The boundary conditions used for simulating infiltration to a fixed water table were as follows

Upper boundary condition

\[-K(h) \left( \frac{\partial h}{\partial z} + 1 \right)_{z=0} = q \quad (2)\]

Lower boundary condition

\[ h(z, t)_{z=-L} = 0 \quad (3) \]
Initial condition

\[ h(z, 0) = h_0(z) \quad -L < z < 0 \]  

where \( h_0(z) \) is some distribution of pressure head in the column.

**COMPUTER RESULTS**

The porous material used in the numerical studies was no. 17 sand. The \( h(\theta) \) and \( k(\theta) \) relationships of the No. 17 sand are given in Fig. 1. As shown in Fig. 1 at pressure head of zero the sand is fully saturated (\( \theta_{\text{sat}} \)). With decreasing pressure the soil remains saturated until the capillary force maintaining saturation are overcome and the largest pore drains. The negative pressure at which this occurs is termed the air entry value. A further decrease in pressure head below the air entry value results in decreasing in moisture content. This continues until the residual moisture content is reached (\( \theta_1 \)), where the sand stops draining and the remaining water is held primarily as water films around the sand grains and in angles between sand grains. The saturated hydraulic conductivity and saturated water content are 0.8 cm/min and 0.292 respectively.

Figure 2 and Figure 3 show the pressure head and water content profiles for the case where the rainfall rate is -0.95 cm/min. As infiltration proceeds, a pressure head front moves downward into the column. The front becomes less and less steep as it moves down the column. The pressure head at the top of and at each point in the column increases and approaches a steady state limiting value. After a finite time of 15 minutes the flow in the column is steady state and the pressure head value is -9 cm. As rainfall continues the pressure head increases and becomes zero along the soil profile at \( t=20 \) min. Then the only head that exists is the elevation head (which is always constant) and responsible for the unit hydraulic gradient.
Thus, in the absence of vertical hydraulic gradients, the addition of very small amount of water will relieve the capillary tension and cause an almost instantaneous rise in the water table to ground surface. It should also be noted that with the loss of small amount of water, either by downward drainage or by evaportranspiration, the water table would return quickly to its original position.

**UNSATURATED SHEAR STRENGTH PARAMETERS**

Early works Bishop (1960) on unsaturated soils where pore air and pore water pressure are accounted for has led to the establishment of the following relationship.

\[ \sigma' = (\sigma - h_a) + X(h_a - h_w) \]  

where

- \( \sigma \) is the total normal stress
- \( h_a \) is the pore air pressure
- \( h_w \) is the pore water pressure
- \( X \) is an empirical parameter which depends on the saturation ratio.

As \( X \) is difficult to determine experimentally very limited use is made of the relationship. Frelund and Morgenstern (1977) overcome the difficulty by proposing independent stress variable \( -h_a \) and \( (h_a - h_w) \). The unsaturated shear strength is then given by

\[ \tau = C' + (\phi' - h_a) \tan \phi + (h_a - h_w) \tan \phi^b \]  

where

- \( C' \) is the effective cohesion of the soil
- \( h_a - h_w \) is the matrix suction
- \( \sigma - h_a \) is the total stress with respect to air pressure
- \( \phi' \) is the effective angle of internal friction
- \( \phi^b \) is the angle of shear strength increase with respect to matrix suction.

Thus, it can be seen that the matrix suction is adding to strength of the soil, apparently increasing the cohesion of the soil. The normal shear strength equation may be written,

\[ \tau = C + (\sigma_a - h_w) \tan \phi \]  

It can be seen that comparing equations 6 and 7.

\[ C = C' + (h_a - h_w) \tan \phi \]
Equation 8 has two distinct advantages; a- there is a smooth transition from undaturated to saturated soil conditions as $h_a$ becomes equal to $h_w$, the equation takes form of equations 7. b- The shear strength can easily be incorporated in the normal slope stability as for saturated soil.

Fig. 2: $h(Z)$ Profiles for homogeneous column of no. 17 sand during infiltration rate ($0.95 \text{ cm/min}$) with stationary water table. The numerals on the curves represents the time in minutes from the start of rainfall.

Fig. 3: $\theta(z)$ Profiles for a homogeneous column of no. 17 sand during infiltration rate ($0.95 \text{ cm/min}$) stationary water table. The numerals on the curves represents the time in minutes from the start of rainfall.
Modifications can be made to the conventional triaxial test to allow shear strength parameters to be determined at various values of suction. Details of set up is given by Ho and Fredlund (1982). Sweeney and Robertson (1979) have established that the relationship between the cohesion with pressure head varies linearly, as shown in Fig. 4.

Based on this assumption, a linear relationship between the cohesion and angle of internal friction were assumed to vary linearly with the pressure head as shown in Figs. 5 and 6.
The value of C has been arbitrarily chosen to vary from a value of 30 kPa to 0 kPa over the range of pressure head experienced by the soil during infiltration time.

The internal angle of friction, $\phi$, is also assumed to vary linearly with respect to pressure head. When pressure head increases, the confining effect due to suction will be reduced and the value of $\phi$ will increase. At groundwater table, the cohesion becomes zero and hence the soil is virtually cohesionless. Hence a value of $\phi$ equalling the slope angle, $\alpha$, is chosen.

EFFECT OF PRESSURE HEAD ON THE STABILITY OF SLOPES

The Bishop's method of slices has been adopted to demonstrate this effect. Calculation has been performed at different depths along the soil profile for a period of 20 minutes. It can be seen in Fig. 7 that at 20 minute for all depths, the slope will be at the verge of sliding. The reduction in the factor of safety, $F$, is very drastic for shallow depths. At depth of .1 m the factor of safety reduces from a value of 56 to a value of 1 in 20 minutes.
At the capillary fringe zone the reduction in $F$ is not that drastic and hence the infiltration have little effect on pressure head and consequently the factor of safety. This is due to that this zone has little or no storage capacity of water and remains saturated under negative pressure head. Knowing when and where the failure surface will be located, this would help the Geotechnical engineer to provide modifications to their designs to avoid the causes of such drastic changes. Design charts can be established using dimensionless parameters such as $h/h_0$, $z/z_0$ and time factor.

CONCLUSION

The incorporation of soil shear strength analysis describing the behaviour of changing effective stress with time into a computer-based numerical solution of the equation for the flow of water in an unsaturated porous material enables one to predict when the failure will occurs.

REFERENCES


[6] Sweeney D. and Robertson P. (1979), "A fundamental approach to slope stability in Hong Kong", Hong Kong Engineer. 7 No. 10, 35-44